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THESIS

THE NAVAL OFFICER ASSIGNMENT PROBLEM

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ABSTRACT

The effectiveness of the officer distribution system of the Navy is strongly dependent on the assignment officers' daily assignment decisions. The officer assignment problem is to determine the optimal assignment of officers to billets on a continuing time basis. A procedure is developed in this study by which ranked assignment alternatives can be provided the assignment officer to assist him in making his decisions. The ranking of the alternatives is based on an index or value measure developed from the quantifiable assignment information. The assignment alternatives are developed for a specified assignment period of interest and represent trade-offs between feasible assignments and times of assignment. The procedure makes long range assignment planning feasible by reducing the problem to one of manageable size for the assignment officer's consideration.

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CHAPTER I

INTRODUCTION

A. Officer Distribution System:

The effective assignment and reassignment of officers to billets in the U. S. Navy is a complex and dynamic decision problem involving multiple objectives. Centralized distribution control of all naval officers is maintained by the Bureau of Naval Personnel (BuPers). Such control permits assignments based on the entire officer inventory.

All officer assignments are determined within the Officer Distribution Division of BuPers. The Officer Distribution Division of BuPers is organized into three major branches, the Grade Assignment Branch, the Officer Placement Branch, and the Staff Corps Liaison Branch. This study will only consider the distribution of the unrestricted line officers who comprise more than 70 percent of the total officer inventory. Distribution of the line officer is effected by the Grade Assignment and Officer Placement Branches. The Grade Assignment Branch is subdivided into individual Rank Assignment Sections which represent the officer at the Bureau level. The officers within these sections will be referred to as assignment officers or detailers. The Placement Branch, on the other hand, is the representative of the individual Naval activities and Commands at the Bureau level. This organization insures that the individual officer and naval activity concerned are represented in each assignment decision.

Naval officers are reassigned to different duties periodically throughout their careers. Such rotation is required both for professional development and to reunite families after long periods of separation. A typical Professional Development Pattern for code 1310 officers is included as Figure 1. Such patterns reflect the desired progression to increased responsibilities throughout the officer's career and the sea/shore rotation patterns demanded by the division of the Naval establishment into a land-based "shore establishment" and sea-based "fleet". Such patterns are general in nature and are used as a guide in planning future assignments.

When an officer is assigned to duty, his assignment is for some specified period or tour length. At the time of his reassignment a projected rotation date (PRD) is established for regular Naval officers and either an expected loss date (ELD) or release from active duty date (RAD) for reserve Naval officers. Such dates reflect when the officer can next normally expect orders and are established for a variety of reasons such as completion of a specified overseas tour, special training, a contractual obligation, obligated active duty, etc.

The assignment cycle is based on the individual rotation, release and expected loss dates established for each officer. To the various placement officers such dates signify anticipated vacancies in the units under their cognizance. Approximately one year prior to such an anticipated loss, the

FIGURE 1

GR*	YEARS	PERIOD	PHASE	ASSIGNMENT AREAS		
ENS	1–2	FUNDAMENTAL PROFESSIONAL DEVELOPMENT	Basic Trng	FLIGHT TRAINING		
LTJG	2–4	FUNDAMENTAL PROFESSIONAL DEVELOPMENT	First Operational	FLEET TRAINING SEA Operational Squadrons		
LT	4–9	FUNDAMENTAL PROFESSIONAL DEVELOPMENT	Educational	SHORE Training Command Postgraduate Education Undergraduate Program Bureaus, NAS, OPNAV, TPS CRAW		
LCDR	9–15	INTERMEDIATE PROFESSIONAL DEVELOPMENT	Second Operational	FLEET TRAINING SEA Split Tour Squadron/Ship/Staff/Overseas		
LCDR	13–16	INTERMEDIATE PROFESSIONAL DEVELOPMENT	Subspecialty and/or Educational	SHORE Training Command Postgraduate Education Undergraduate Program	Service College Subspecialists normally assigned to subspecia' y field OPNAV, Bureaus, NAS, USNA.	
CDR	15–21	INTERMEDIATE PROFESSIONAL DEVELOPMENT	Operational Command Development; Subspecialty and/or Educational	FLEET TRAINING SEA Split Tour Squadron/Ship/Staff/Overseas	Assignments varied as necessary to round out qualifications and meet service needs	SHORE Planning and administrative duties in aviation activities Research and development OpNav, Bureaus Training Command Service College
CAPT	21–30	ADVANCED PROFESSIONAL DEVELOPMENT	Final Development	Deep draft and major command Staff Overseas	Assignments varied as necessary to round out qualifications and meet service needs	Policy planning and managerial duties in OpNav, Bureaus, technical offices, shore commands, and naval districts (may be allied to subspecialty field) Research and development activities Joint staffs Service colleges CO of shore activities

* Indicates average promotion points. Actual promotion point will vary within each promotion zone and will shift according to required promotion flow.

PROFESSIONAL DEVELOPMENT PATTERN FOR CODE 1310 OFFICERS placement officers notify the appropriate rank desks that a replacement is desired. Such a process is known as "posting".*

Each grade assignment officer is, therefore, aware of the billet assignments to which the officers under his cognizance may be ordered and the officers available for such assignment. Officers are normally considered for those billets available within a one - two month period of their rotation dates. The assignment officer reviews each officer's records, past experience, preference and data card. It should be noted that the assignment officer is in a position to compare the qualifications of each officer with those of his contemporaries who are also available for new assignments.

On the basis of such comparisons, the known billets, and career developmental patterns, the assignment officer determines each officer's next assignment, establishes a new rotation date and nominates the officer for the assignment to the appropriate placement desk. If the officer is approved for the assignment by the placement desk, orders are prepared and issued. If disapproved, the officer is considered and nominated for other billets and the assignment process is repeated.

*The notification is forwarded on a "posting strip" designed to clearly display billet requirements on the assignment officer's posting board.

B. The Assignment Decision

While the procedures used by each individual assignment officer in making assignment decisions probably vary with the individual, it is clear that all assignment officers have essentially the same type of assignment problem. At any particular time, he has a list of the officers available for reassignment and a list of billets to which they may be assigned. His objective is to make the "best" possible assignment decisions.

In the literature problems of optimal assignment have received considerable attention (3). The classical statement of this problem has been to determine the optimal assignment of m individuals to m tasks where it is assumed that there are measurable differences in each individual's ability to perform different tasks and between individuals in performing the same task.

An illustration from King (6) might serve to clarify the structure of the classical problem, and illustrate the nature of the assignment decision. Consider the assignment of four individuals to four tasks such that each individual is assigned to only one task and each task is assigned to only one individual. The predicted hours required for each individual to perform each task, if assigned, are displayed in Table 1. The objective is to determine that assignment which minimizes the total man hours required to complete all tasks.

TABLE 1

Tasks

Individuals:		T_1	T_2	T_3	T_4
	I_1	1	2	3	3
	I_2	4	3	3	2
	I_3	1	4	5	5
	I_4	4	3	7	6

Assignment problems are normally displayed in a matrix such as Table 1. Such a matrix will be referred to in this paper as the "value" matrix and the individual elements of the matrix as the "assignment values".

Two intuitive decision rules one might consider for solving this problem are: (1) "assign each individual to the task he performs best", or (2) "assign each job to the individual who does it best".

Consider the first rule. Application of the rule is impossible since I_1 and I_3 each perform T_1 in one hour and T_1 is the task each does best. Since only one man can be assigned to each task, it seems logical to consider the tasks performed "second-best", T_2, I_1 can perform it in 2-1=1 additional hours compared with 4-1=3 additional hours for I_3. I_3 is therefore assigned to T_1 and I_1 to task T_2. But T_2 is the task performed best by I_4. "Quite quickly, one realizes that this innocuous-appearing problem is quite complex." (6, p.5) The authors note that similar problems occur using the second intuitive rule. Better decision rules are obviously needed and have been developed in the literature.

The general m x m classical assignment problem of this type can be more formally stated as follows:

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{m} a_{ij} y_{ij} \qquad \text{(I-1)}$$

$$\text{subject to} \quad \sum_{i=1}^{m} y_{ij} = 1 \quad \text{for all } j, \qquad \text{(I-2)}$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad \text{for all } i, \qquad \text{(I-3)}$$

$$\text{and} \quad y_{ij} = 0,\ 1 \quad \text{for all } i \text{ and } j. \qquad \text{(I-4)}$$

It should be noted that the matrix $A = (a_{ij})$ is a "value" matrix similar to Table 1. The solution to this problem can be found by enumerating all twenty-four possible assignments or more easily by using the algorithms to be discussed in Chapter II. Table 2 displays all possible solutions to the problem and is included to define terms to be used in this study. It is noted that the "value" matrix in this example implied formulation as a minimization problem. However, maximization problems can be readily transformed so that minimization algorithms are applicable.

The complexity of the assignment officer's task and the two phase nature of the problem is apparent. In order to make the "best" possible assignment decisions, the assignment officer must first estimate or place a "value" on each possible assignment, and then make the actual assignments in a manner which will best achieve the Navy's objectives.

Since assignment officers do make assignment decisions an "assignment value" does exist implicitly, if not explicitly. It is apparent that any solution procedure developed must assign some quantitative value to each possible assignment. While the formal development of a "value" measure will be delayed to Chapter III, it is noted that the assignment decision is based on both quantifiable information such as qualifications, demonstrated performance, past experience, etc., and the assignment officer's experienced judgment. The assignment value developed in Chapter III will be derived from quantifiable assignment information.

For the $m \times m$ symmetric problem under discussion, there are m! possible assignments. This means, for example, that there are 10! or 3,628,800 possible assignments when ten officers are considered for ten billets. Assignment problems of such size often face assignment officers. To enumerate all possible permutations is not very practical even on present-day computers.

In developing a solution procedure for the officer assignment problem it seems clear that some set of "best" solutions are desired since the solution obtained will only be optimal on the basis of the quantifiable assignment information utilized. Such solutions will be referred to in this study as the "numerically preferred solutions". For example, in Table 2, A_1 is numerically preferred to A_2, and solutions A_1 and A_3 are numerically preferred to A_8. Such solutions can provide a decision basis for the assignment officer's consideration. Because of the nature of the military profession and the possibility of unforeseen attrition, flexibility in the solution procedure must also be obtained.

TABLE 2

	T_1	T_2	T_3	T_4	Value
A_1	I_3	I_4	I_1	I_2	9
A_2	I_3	I_4	I_2	I_1	10
A_3	I_1	I_4	I_3	I_2	11
A_4	I_1	I_4	I_2	I_3	12
A_5	I_3	I_1	I_2	I_4	12
A_6	I_3	I_1	I_4	I_2	12
A_7	I_4	I_1	I_3	I_2	13
A_8	I_4	I_3	I_1	I_2	13
A_9	I_1	I_3	I_2	I_4	14
A_{10}	I_1	I_3	I_4	I_2	14
A_{11}	I_4	I_1	I_2	I_3	14
A_{12}	I_3	I_2	I_4	I_1	14

	T_1	T_2	T_3	T_4	Value
A_{13}	I_4	I_3	I_2	I_1	14
A_{14}	I_1	I_2	I_3	I_4	15
A_{15}	I_2	I_4	I_3	I_1	15
A_{16}	I_4	I_2	I_3	I_1	15
A_{17}	I_1	I_2	I_4	I_3	16
A_{18}	I_3	I_2	I_1	I_4	16
A_{19}	I_4	I_2	I_1	I_3	16
A_{20}	I_2	I_1	I_3	I_4	17
A_{21}	I_2	I_3	I_1	I_4	17
A_{22}	I_2	I_4	I_1	I_3	17
A_{23}	I_2	I_1	I_4	I_3	18
A_{24}	I_2	I_3	I_4	I_1	18

The optimal assignment planned six months in advance might vary considerably prior to issuance of orders.

It has been shown that at any given time the assignment officer's problem is the "optimal assignment problem" of linear programming. However, the officer assignment problem is to determine the optimal assignment of officers to billets on a continuing time basis. Since officers are only considered for billets available in a time frame around their rotation dates, it is clear that a billet could be available for which no officer then available was qualified, or that an officer uniquely qualified for a particular billet is available and the billet isn't. Reassignment in this context implies utilization of the officer to the best extent possible.

It is important to note that the method of determining when an officer is available for reassignment defines the set of possible billets to which he can be considered for reassignment. Where he can be assigned and how his qualifications may be utilized is therefore a function of his scheduled reassignment time. Since an officer's assignment to duty is normally for some specified tour, it is evident that the assignment decision is not unlike an investment decision and to some extent implies that the assignment is the preferred investment of his particular talents during the specified period.

It is evident that some reassignment availability system such as that presently employed is needed to ensure orderly rotation. Intuitively however, more effective utilization is possible if officers could be considered for more billets, i.e., for some greater time span about their projected rotation dates. In other words, it may be profitable to delay reassignment, or assign early, if more preferred assignments could be achieved. In a limited sense this presently is done when officers are extended in their units or ordered out early in order to meet special training dates. The major emphasis in the development in Chapter II will be to provide a solution procedure of the officer assignment problem on a continuing time basis which will satisfy both objectives.

C. Purpose

The purpose of this study is to analyze the officer assignment problem and develop a solution procedure which will assist the assignment officer in making assignment decisions. The complexity of the assignment decision has been described and those desirable characteristics of the solution procedure defined. Major characteristics desired include:

(1) The derivation of any desired set of numerically preferred solutions given the officers to be assigned and the billets available;

(2) Solution of the officer assignment problem on a continuing time basis;

(3) Flexibility in the solution procedure to respond to a varying assignment environment.

CHAPTER II

DEVELOPMENT OF THE ASSIGNMENT SOLUTION PROCEDURE

A. Introduction

In this chapter one will find an assignment method, a sequential solution procedure, that has characteristics intermediate to present day assignment practices and the solution procedure developed throughout the remainder of this chapter. This sequential solution procedure is included to illustrate combinatorial problems and provide a vehicle to transition from present day assignment methods to the authors' solution procedure. Also, one will find the manner in which the analysis proceeded to determine that the out-of-kilter algorithm of Ford and Fulkerson[5] was most appropriate to solve the officer assignment problem. Examples will demonstrate the important features of the out-of-kilter algorithm, with regard to tractability and flexibility in various assignment situations. An iterative procedure is then developed utilizing the out-of-kilter algorithm to proceed from the numerically preferred solution to all lesser numerically preferred solutions; and then a method is developed to find the total amount of extension time needed to execute each solution.

B. Sequential Solution Procedure

Assume that a detailer has approached the personnel assignment problem armed with only the algorithm to solve the classical transportation problem as posed and solved by Frank L. Hitchcock. He can use this method in applying the transportation algorithm to sequential type solutions. An example of this approach and conclusions drawn follow.

Because of its importance in the following discussion we first take a moment to look at the effect of varying rotation dates. Consider the following model. In this simple model we allow officer (i), denoted by O(i), to be available for reassignment for some period of time Δt_j about his projected rotation date, for example:

let Δt_j $\equiv$ maximum length of time O(i) will be considered for reassignment at the end of his tour in billet (j)

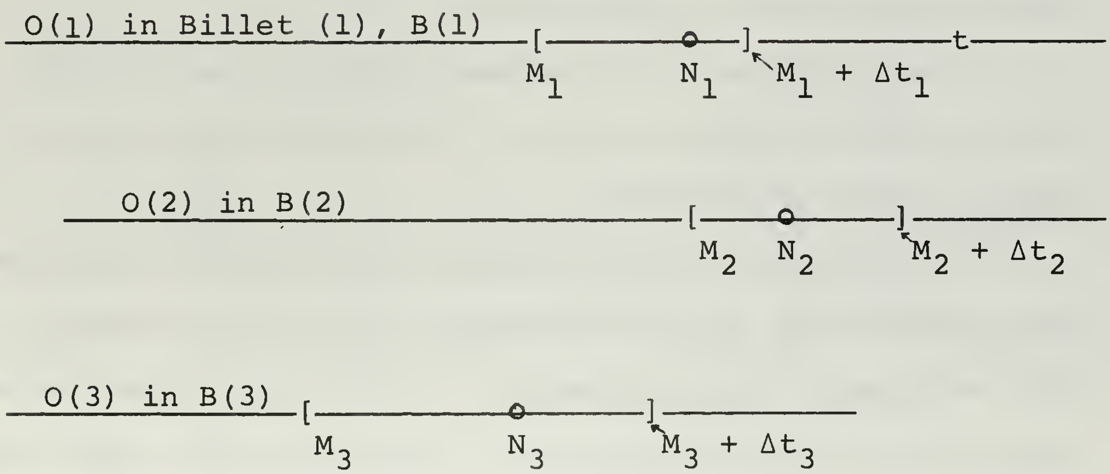
N_j $\equiv$ "normal" tour length, i.e., 2 years, 3 years, etc., for a specific billet (j)

M_j $\equiv$ the minimum tour length for a specific billet (j)

where $M_j \leq N_j \leq M_j + \Delta t_j, \forall_j$

Suppose we have three officers with the following tour profiles:

FIGURE 2



As the figure is drawn, O(1) could be assigned to either B(2) or B(3). If however, $M_2 > M_1 + \Delta t_1$ and $M_1 > M_3 + \Delta t_3$ then O(1) could not be assigned to either of the other two billets. Note also that if $M_2 \leq M_3 + \Delta t_3 \leq M_2 + \Delta t_2$ then O(3) could be considered for B(2).

From Figure 2 it is easy to see how using extensions in assignment planning makes possible assignments that formerly might not have been feasible.

To facilitate the description of the sequential approach a detailer might pursue, we will use the tour profile shown in Figure 3 and the table of assignment values given in Table 3 as an example.

The following assumptions and guidelines will be used in the study of the sequential assignment approach.

(1) An officer is required to be reassigned at some point between his min ([) and max (]) tour dates inclusive.

(2) The transportation algorithm is used at each month (1, ..., 7) to determine the total value of reassigning officers available in that month.

(3) A decision is not actually required until some officer reaches his max tour (a "critical month", e.g., officer one at month four).

(4) A decision is made based on the lowest-value assignment available up to and including the "critical month". This assignment must necessarily involve the officer whose max tour is associated with the "critical month" under consideration.

FIGURE 3

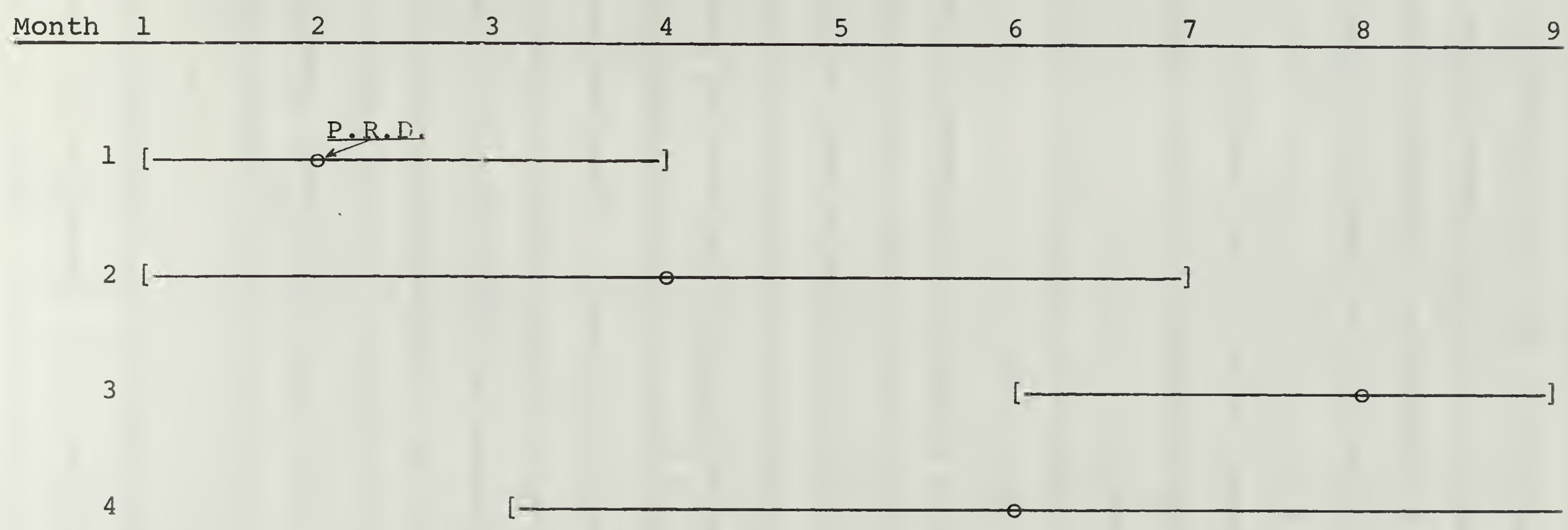


TABLE 3

ASSIGNMENT VALUE MATRIX

	B(1)	B(2)	B(3)	B(4)
0(1)	1	2	3	3
0(2)	4	3	3	2
0(3)	1	4	5	5
0(4)	4	3	7	6

e.g.

$a_{12} \equiv$ value of assigning officer 1 to billet 2 = 2

$a_{43} \equiv$ value of assigning officer 4 to billet 3 = 7

etc.

(5) It was assumed the assignment officer did not have access to all possible solutions, and hence lacked the capability to compare all possible total assignment values. Admittedly, in this small example the assignment officer could list all such solutions, but not readily in even a slightly larger problem.

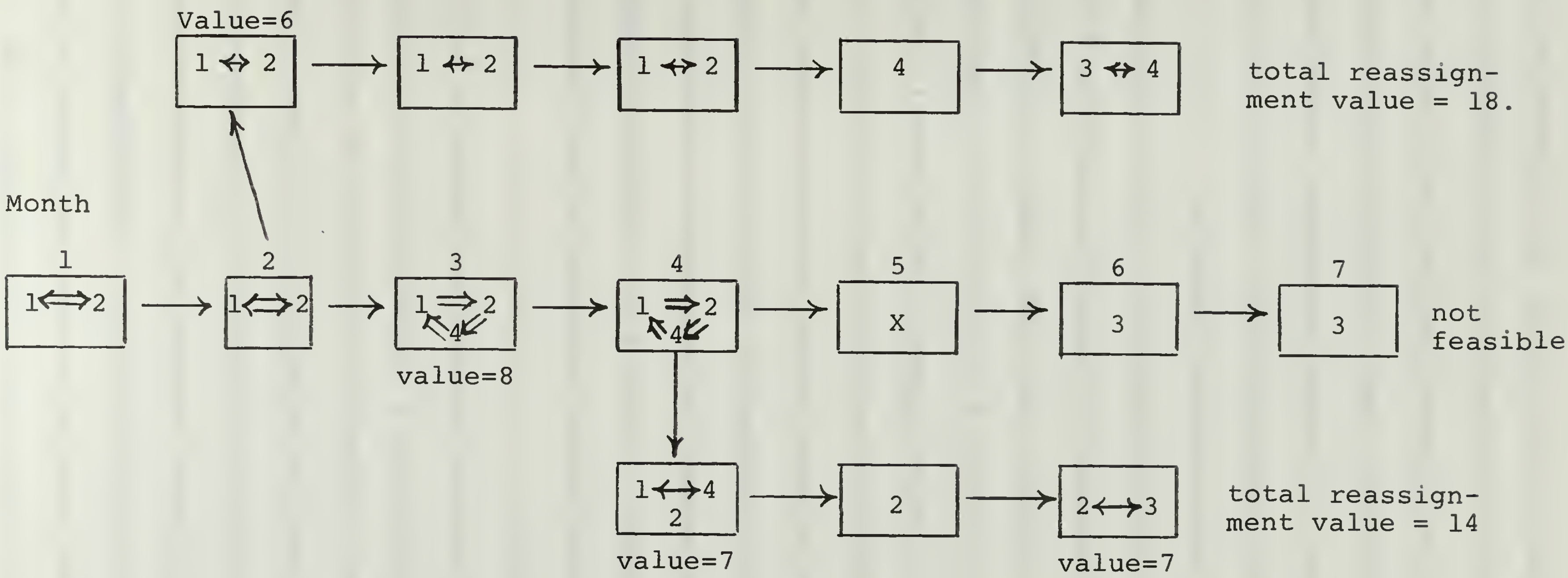
(6) The total assignment value of each solution is found by summing the values associated with each assignment that is made in that solution.

(7) With no loss in generality, assume O(1) is presently filling B(1), O(2) is in B(2), etc.

Figure 4 illustrates a sequential assignment process the detailer might follow if he was to make the best use he could of the transportation algorithm at each decision time (critical months) as conceived and applied by the authors and the resulting solutions. The detailer must make a choice of some sort on or before the fourth month because O(1) reaches his max tour in that month. Therefore he begins by calculating the value of assignments in each of the months 1, 2, 3 and 4. Suppose he chose (1 $\leftrightarrow$ 2) because it was least value (6 vice 8) up to and including month four. He would then proceed as indicated by the top chain in Figure 4. Subsequently he would make the (3 $\leftrightarrow$ 4) assignment in month six. This is a feasible solution with a total reassignment value of 18.

If the detailer would have had the protracted information at his disposal to allow him to determine all feasible alternatives, he would have undoubtedly chosen the solution

FIGURE 4



for example:

(1 ⟺ 2) means the transportation algorithm applied in this month gave as the solution "switch officers 1 and 2"

(1 ⟷ 2) means that the solution has been executed at this point in time

with the total assignment value of only 14, ($1 \leftrightarrow 4$, $2 \leftrightarrow 3$). In defense of our hypothetical detailer, it is not obvious at first glance even in this small problem that if the detailer had switched O(1) and O(4), as in Figure 4, at a value of 7 at month four, that the excess assignment value incurred there would be more than compensated for in month six because ($2 \leftrightarrow 3$) is an option preferred to ($3 \leftrightarrow 4$).

From this example it is easy to see that as the number of officers in a time horizon increases the number and length of possible chains increases. Also the number of chains terminating in infeasible solutions increases. In short, the combinatorial aspects of this approach would make it an unwieldy tool to use even if a computer were available.

A desirable method appears to be one that retains the merits of the procedure discussed to this point and circumvents the necessity of recounting all possible solutions to attain the most preferred. It would be a generous bonus, too, if this sought-after method could efficiently offer up successive best solutions.

It would be well to note in general, referring again to Table 2, that there exists a unique preferred solution (A_1) to the assignment of these four officers if each reassignment period were extended through all 9 periods. There also exists a unique preferred solution (A_2) for these officers if assignments could be made only one month either side of their projected rotation dates. The assignment value of A_1 is less than or equal to the assignment value of A_2 since the constraints have been relaxed in A_1. The assignment value of the preferred assignment to the problem as it stands in Figure 3 is greater than or equal to assignment value of A_1 and less than or equal to the assignment value of A_2 since it has constraints intermediate to the hypothetical limit constraints. In other words, if all the officers in a time horizon could be reassigned at any month, the resulting preferred solution would be the best attainable under the model as it is presently structured. This concept is, of course, unrealistic when you are talking about actual assignments because other constraints must be considered also. For example, career patterns impose general sea and shore tour lengths. Cost and time associated with transfers and training might well be a consideration. These are merely a few of the many such important considerations which must be taken into account in any real world assignment problem.

As has been stated, the authors believe that the correct assignment policy lies somewhere between complete freedom in assignment and reassignment close aboard the projected rotation date. The method which will traverse this middle ground should not jeopardize career rotation patterns as outlined in the Officers Fact Book[9] but still provide wider choices of billets for officers. It should not be misrepresented as being capable of giving the best solution under all real world constraints, for that in total, can only be determined by the assignment officer based on his knowledge of many subjective factors. It should yield preferred solutions based on those variables which are quantifiable and hence free the assignment officer to ponder the subjective aspects of the problem. As a consequence of these observations, it would appear that an approach which is based on billet qualification variables (i.e., flight time, etc., which are explained in detail in Chapter III), as the quantifiable part and the amount of extensions allowed for each billet as the subjective part might be reasonable. By way of illustration, suppose the example problem was solved as an unrestricted assignment problem (the time ignored). The transportation algorithm would generate the optimal assignment (1 $\leftrightarrow$ 3, 2 $\leftrightarrow$ 4) with a value of 9. This solution is not possible though if the tour profile in Figure 3 is adhered to. <u>If</u> only O(1) could be extended two months or <u>if</u> only O(3) were available two months earlier, or <u>if</u> both of their periods could be made to meet at month five, this solution could be effected. Only the assignment officer can decide whether or not any of these <u>ifs</u> are possible and, if so, which one.

If the value matrix were as in Table 4 the unrestricted numerically preferred solution would be 1 $\rightarrow$ 2 $\rightarrow$ 3 $\rightarrow$ 4 $\rightarrow$ 1. To realize why this chain type solution would pose difficulties, one need only consider O(1), O(2) and O(3) in Figure 3. Send O(1) to relieve O(2) in month four and O(2) to relieve O(3) as soon as possible which is month six. This requires that B(2) has O(1) and O(2) in it for two months and if the solution is completed, billet four is empty in months four and five. Here again, the numerically preferred solution has implied a solution which is not possible if the tour profile is strictly adhered to. It is obvious that any attempt to execute this solution with the given tour profile will terminate with at least one gapped billet (i.e., a billet with no one filling it) and at least one billet that for some time has two officers assigned. And as before the assignment officer is the best qualified to decide whether properly stretching the reassignment periods is worth the associated gain.

TABLE 4

	B1	B2	B3	B4
O1	6	2	5	4
O2	7	8	1	2
O3	5	6	4	2
O4	3	6	4	9

Either of the three <u>ifs</u> in the previous example would allow this solution to be effected also.

The subjective aspects of the solution method to be developed are presented later in this chapter. However, a few points should be mentioned now. They are:

(1) In a chain type solution all officers concerned must be reassigned in the same month to assure there are no gapped billets. This feature will be retained in the solution method to be developed.

(2) The detailer is the person qualified to decide if any extensions are tolerable and if so, to which billets and how much of an extension can be allowed to each reassignment period. A similar concept will also be used extensively in the solution method.

(3) The unconstrained numerically preferred solution is a function only of the numbers in the assignment value matrix because it is independent of whatever slack might be allowed for reassignment about an officer's projected rotation date. Such a solution would seldom be compatible with the time constraints of finite reassignment periods.

C. The Assignment Model as a Network

Because the classical assignment model is but a special form of a transportation model a graph theoretic formulation can be used which results in the assignment model being described as a network involving flows and costs. With such a formulation of the model, the unconstrained assignment problem can be solved as a <u>minimum</u> <u>cost</u> <u>flow</u> problem by the out-of-kilter algorithm of Ford and Fulkerson[5]. The use of this algorithm provides a very efficient means of solving for a series of numerically preferred solutions. We will consider how the algorithm is to be used after we show how the assignment model can be formulated as a network.

Officers who have projected rotation dates within the time horizon under consideration are represented by the nodes O(1), O(2), O(3), and O(4) on the left side of Figure 5. Similarly billets B(1) through B(4) are on the right side of the same figure. An arc is drawn from each officer to every billet for which he may be considered. The arcs are labeled in the manner described below for arc (O1, B2).

Each arc is labeled with the appropriate value of assignment from Table 3, (e.g., O1 → B2). The labels on the nodes
(, 2)

FIGURE 5

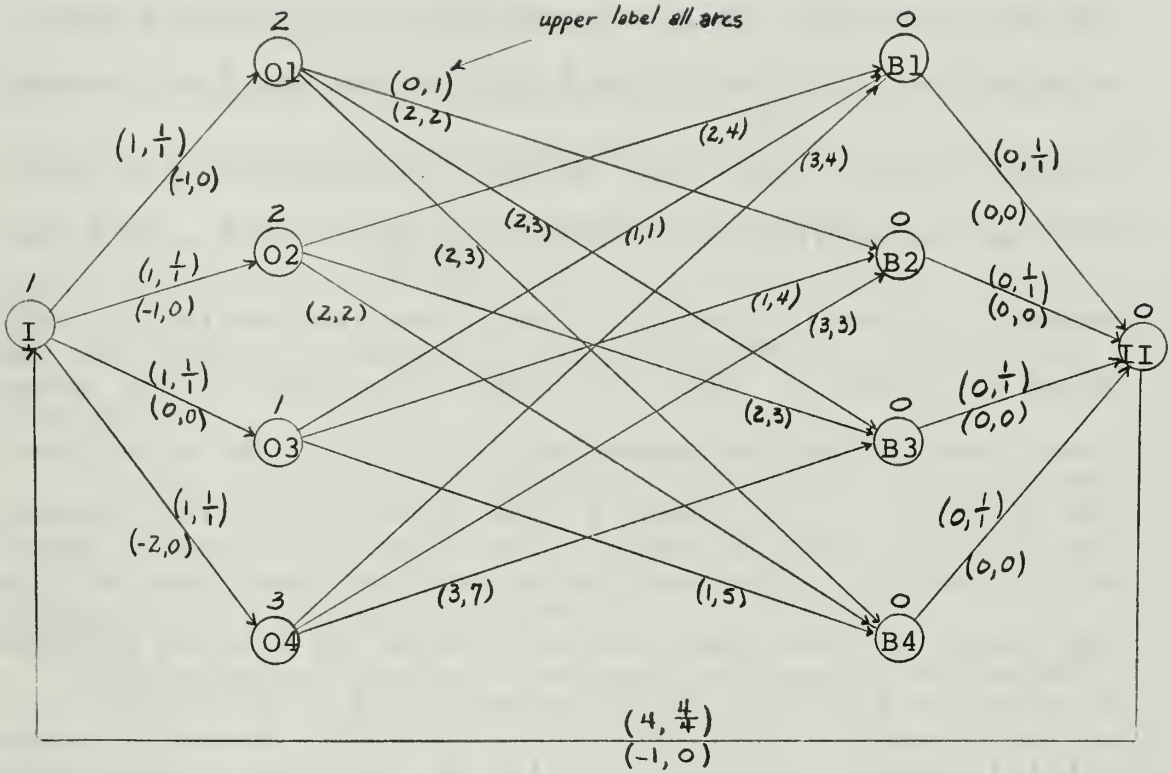
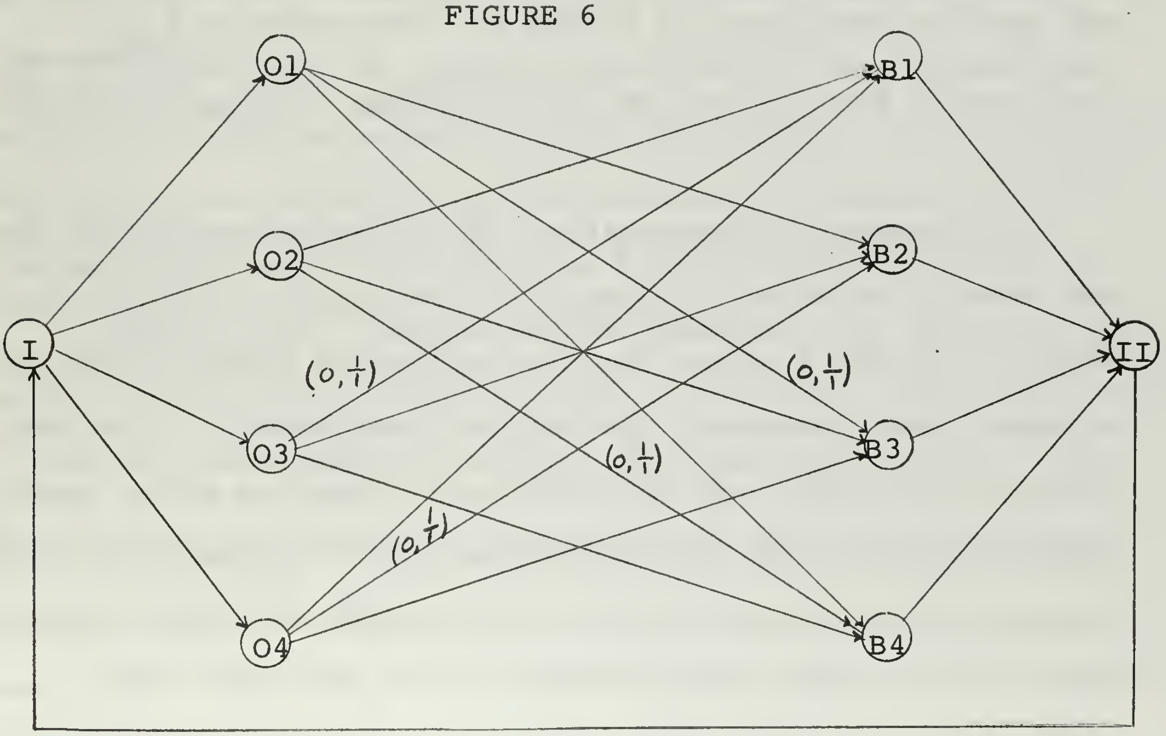


FIGURE 6



are v_i's calculated using the minimum route algorithm of Ford and Fulkerson[5]. This method seems expeditious even though the out-of-kilter algorithm allows starting with any set of v_i values. The corresponding $(v_i - v_j)$ values were assigned (e.g., $O1 \xrightarrow[(2,\ )]{} B2$) and the min and max flow allowed through the arc, (e.g., $O1 \xrightarrow{(0,\ 1)} B2$). The arc (O1, B2) then carries the notation in the form $\xrightarrow[(2,2)]{(0,1)}$. The arcs from source I to the Oi (i=1, ..., 4) nodes are labeled as indicated in Figure 5. The only number that will differ among these arcs is the corresponding $(v_i - v_j)$. The arcs from the Bj (j=1, ..., 4) nodes to the sink II are also labeled as in Figure 5. The notation on each of these arcs will be the same. The arc from sink to source was labeled $\xrightarrow[(-1,0)]{(4,4)}$ to ensure a flow (the reassignment) of four officers. An initial feasible circulation flow as required by the algorithm was used as indicated in Figure 5. The notation $\xrightarrow{\left(\frac{1}{,1}\right)}$ indicates a flow of one out of a maximum allowable flow of one.

The network representation of the assignment model for our example is shown in Figure 5. The optimal solution is shown in Figure 6 where the flow values of unity on the assignment arcs represent the optimal assignments. The arcs (O2, B4), (O3, B1) and (O4, B2) were chosen so as to start with as many arcs in kilter as possible. In completing the feasible circulation flow the first three so chosen required that a flow of one be sent through arc (O1, B3). This last arc is then the only out-of-kilter arc in the initial set-up.

The rest of the out-of-kilter algorithm was then applied and the solution terminated with flow through the arcs as shown in Figure 6. Arc (O1, B3) was brought into kilter and the flow remained as in the initial circulation flow. The value of nine associated with this solution is found by summing the assignment values on each arc with positive flow. Note that this solution is identical to the unconstrained numerically preferred solution obtained by applying the standard transportation algorithm to this problem in Section B.

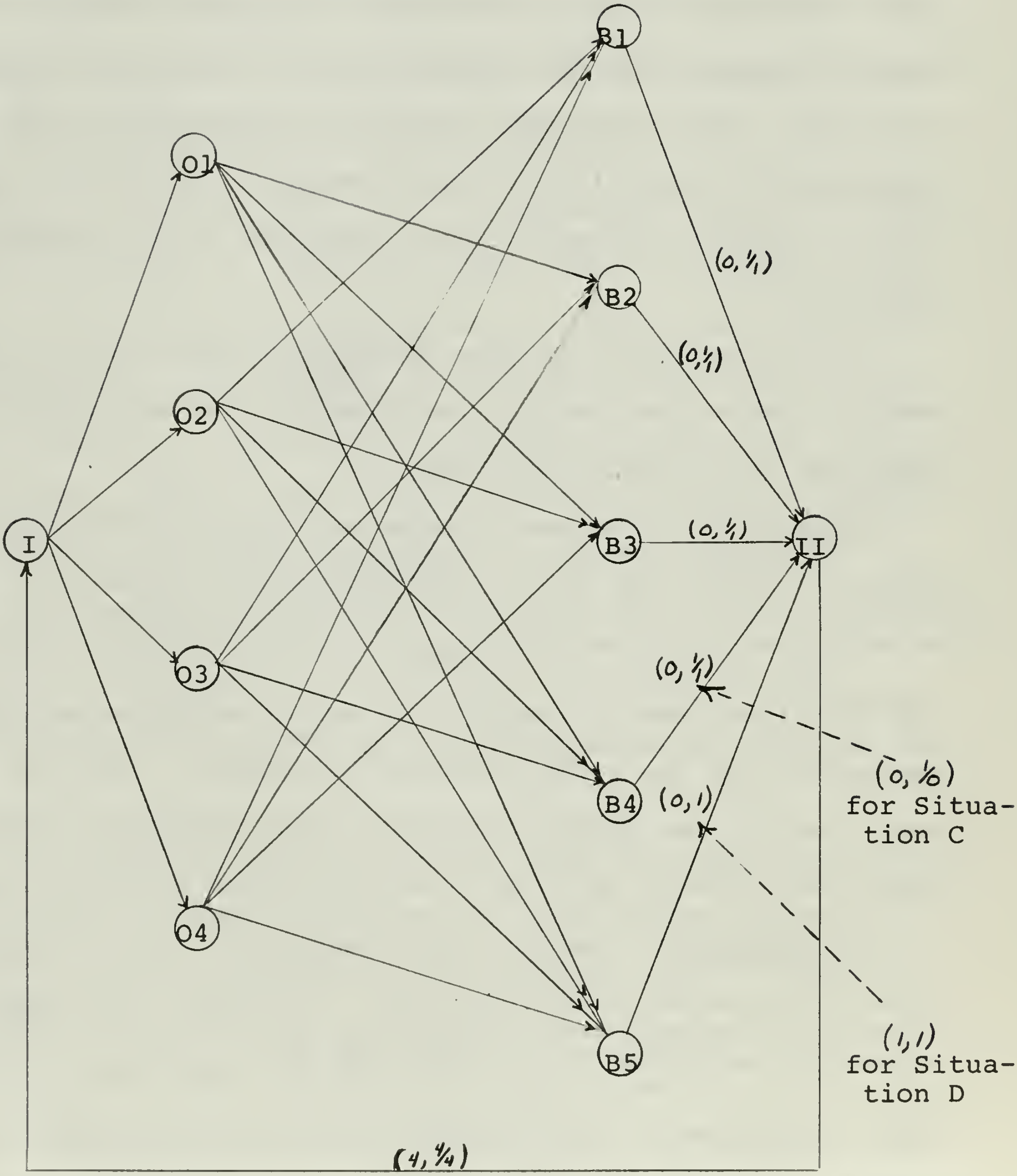
Let us consider now some situations which are not uncommon in detailing and see how flexible the out-of-kilter algorithm is in such situations.

<u>Situation A</u>: Assume a personal situation has arisen and O(1) cannot fill B(3). All that need be done in a case of this sort is to set $M_{O1,\ B3} = 0$ (which makes arc (O1, B3) out of kilter) and to reapply the out-of-kilter algorithm beginning with the solution given in Figure 6. We will iterate in only a few steps to the next best numerically preferred solution; $2 \rightarrow 3 \rightarrow 1 \rightarrow 4 \rightarrow 2$ with a value of ten.

<u>Situation B</u>: Admiral X has requested O(2) be assigned as his aide which is billet three. To ensure this requirement is met by the solution, we need only set $L_{23} = M_{23} = 1$ and again the algorithm will determine the next best solution, $2 \rightarrow 3 \rightarrow 1 \rightarrow 4 \rightarrow 2$.

For situations C and D assume that there is one more billet, B5, as in Figure 7 but only four officers, O(1), O(2), O(3) and O(4) available.

FIGURE 7


B1
O1
B2
O2
B3
I
II
O3
B4
O4
B5
(0, 1/1)
(0,1/1)
(0, 1/1)
(0, 1/1)
(0, 1/0)
for Situation C
(0, 1)
(1, 1)
for Situation D
(4, 4/4)

Situation C: You have just finished finding the solution as shown in Figure 7, and higher authority does away with B(4). There is no need to begin the solution all over. We just put $M_{4II} = 0$, and bring arc (B4, II) into kilter using the algorithm. Note that in a problem where more billets exist than personnel ti fill them $L_{(Bj,II)} = 0, \forall j$.

Situation D: After finding the solution as in Figure 7, word is received that B(5) is aboard a ship now going to Southeast Asia and must be filled. We would just set $L_{(B5,II)} = 1$ and iterate from our present solution to the next best solution.

A summary of the modifications in arc specifications used to iterate from the numerically preferred solution to numerically preferred solutions consistent with restrictions imposed subsequently on the problem follows:

Situation	Method
To make certain:	Set:
A. Officer (i) is not considered for Billet (j)	$L_{ij} = M_{ij} = 0$
B. Officer (i) goes to Billet (j)	$L_{ij} = M_{ij} = 1$
C. Billet (j) is left unfilled	$L_{jII} = M_{jII} = 0$
D. Billet (j) is filled	$L_{jII} = M_{jII} = 1$

The ease with which the above situations are taken care of by the out-of-kilter algorithm can only be fully appreciated when contrasted with the manner in which they would necessarily be handled by someone applying the standard transportation algorithm. When using the transportation algorithm (or any other algorithm of a similar type that the authors have investigated) each situation would require the person to completely resolve the entire problem from scratch after introducing the new restriction. The out-of-kilter algorithm, on the other hand, uses the present solution as the starting point and as a consequence the effort required to get the next solution is considerably less than that required for the standard transportation algorithm.

D. Determination of the Numerically Preferred Solutions

It was intimated in Section B that the out-of-kilter algorithm could give the detailer not only the numerically preferred solution but second, third, etc., best numerically preferred solutions.

It is understandable why someone might raise the question: "Why would anyone want to settle for a lesser preferred solution when the best is available?" The numerically preferred solutions are based solely upon the parameters of the officers and billets that are quantifiable. It is reasonable to assume that in many instances the total amount of extension time needed to effect the numerically preferred solution would be prohibitive. The assignment officer in this case would make the subjective decision to <u>not</u> accept <u>the</u> numerically preferred solution, but rather some solution having a higher assignment value and a tolerable amount of extension time. Hence, the need for successive lesser numerically preferred solutions. Each preferred solution can therefore be thought of as involving a trade-off between total assignment value and extension time.

The method to be presented below for finding successive best solutions eliminates the need for investigating individually all possible assignment combinations. It relies, too, on the important property which allows new solutions to be obtained by beginning the iterations with some existing solution. Basic to success in iterating to next best solutions is the realization that solutions with lower associated values (better solutions already obtained) must be prohibited from returning as the sought-after solution. The latter requirement is fulfilled by disallowing, one at a time, assignments made in the numerically preferred solution. Assignments are "disallowed" by the method used for Situation A, that is, set $L_{ij} = M_{ij} = 0$.

Listed below are the 9 possible solutions* to the 4 x 4 assignment problem posed in Section B.

Possible Solutions:	Value of Solutions:
$a_3\ b_4\ c_1\ d_2$	9
$a_4\ b_3\ c_1\ d_2$	10
$a_2\ b_4\ c_1\ d_3$	12
$a_3\ b_4\ c_2\ d_1$	13
$a_2\ b_3\ c_4\ d_1$	14
$a_4\ b_3\ c_2\ d_1$	14
$a_3\ b_1\ c_4\ d_2$	17
$a_2\ b_1\ c_4\ d_3$	18
$a_4\ b_1\ c_2\ d_3$	18

* $a_2 \equiv$ officer one assigned to billet two

$c_4 \equiv$ officer three assigned to billet four, etc.

a_1, b_2, c_3 and d_4 will not appear since that implies a re-assignment of an officer to the billet he is already filling.

By disallowing a_3 to be in the second best solution the assignment combinations below were the alternatives considered:

$a_2\ b_1\ c_4\ d_3$

$a_2\ b_3\ c_4\ d_1$

$a_2\ b_4\ c_1\ d_3$

$a_4\ b_1\ c_2\ d_3$

$a_4\ b_3\ c_1\ d_2$ ⟵ a candidate for the second best numerically preferred solution.

$a_4\ b_3\ c_2\ d_1$

Then $a_4\ b_3\ c_1\ d_2$ is the solution generated when a_3 is disallowed.

By disallowing b_4 the alternatives listed below were considered:

$a_2\ b_1\ c_4\ d_3$

$a_2\ b_3\ c_4\ d_1$

$a_3\ b_1\ c_4\ d_2$

$a_4\ b_1\ c_2\ d_3$

$a_4\ b_3\ c_1\ d_2 \longleftarrow$ a candidate for the second best numerically preferred solution.

$a_4\ b_3\ c_2\ d_1$

Then $a_4\ b_3\ c_1\ d_2$ is again the solution generated with b_4 disallowed.

By disallowing c_1 the alternatives listed below were considered:

$a_2\ b_1\ c_4\ d_3$

$a_2\ b_3\ c_4\ d_1$

$a_3\ b_1\ c_4\ d_2$

$a_3\ b_4\ c_2\ d_1 \longleftarrow$ a candidate for the second best numerically preferred solution.

$a_4\ b_1\ c_2\ d_3$

$a_4\ b_3\ c_2\ d_1$

Then $a_3\ b_4\ c_2\ d_1$ is the solution generated with c_1 disallowed.

After having disallowed only the first three assignments in the numerically preferred solution when iterating to the second best solution it was noted that all possible combinations have been considered. It will never take more than m iterations to consider all possibilities for the next best solution. In some cases it will take less than m.

To choose the second best numerically preferred solution one need only pick the candidate above with the lowest assignment value. The second best solution is, therefore, a_4 b_3 c_1 d_2 with a value of ten.

The third best solution is found in a manner similar to the second best but now one must prohibit the first <u>and</u> second best from recurring. By simultaneously disallowing a_3 and a_4 to be in the third best solution the assignment combinations below are the alternatives considered:

a_2 b_1 c_4 d_3

a_2 b_3 c_4 d_1

a_2 b_4 c_1 d_3 ←—— candidate for 3rd best

Similarly, by disallowing b_4 and b_3, a_3 b_1 c_4 d_2 becomes a candidate for the third best solution.

Finally, by disallowing c_1, a_3 b_4 c_2 d_1 is an admissable candidate for the third best solution.

The third best numerically preferred solution is then the candidate with the minimum assignment value, i.e., a_2 b_4 c_1 d_3, with a value of 12.

The fourth and successive best solutions are generated in a similar manner by disallowing assignments in more preferred assignments three at a time and so on.

Some may be inclined to think this method too roundabout. Especially when it is obvious from the listing (of all the possible solutions) on page 34 which are the second, third, fourth, etc., best. The authors will be the first to support the latter inclination, but also submit that the method proposed is not devious at all when one considers forty officers instead of four. An assignment problem containing 40 officers has $\frac{39 \times 39!}{2} = 4 \times 10^{47}$ solutions.* Even if the number were reasonable it would be of no value to have generated the $200^{\underline{th}}$ best numerically preferred solution. The first 10 or 20 numerically preferred solutions will hopefully contain the one the detailer selects as his preferred solution.

Since it takes at most (m) iterations to go from an acquired solution to the next least numerically preferred, an <u>upper</u> <u>bound</u> on the number of iterations required to go from the numerically preferred solution to the $p^{\underline{th}}$ best solution is (p-1)(m). All possible assignment combinations could be generated before this iterative upper bound is reached. Consider for a moment the generation of the third best solution on page 36. If c_1 had been chosen to be the first disallowed assignment, we would have scanned six of the seven candidates for third best on the first iteration. The remaining one, $a_2\ b_4\ c_1\ d_3$, then would have been considered on the second when a_3 and a_4 were disallowed. When an assignment, such as c_1, is common to two or more suppressed solutions, it is efficient to disallow that assignment first, making chances better of scanning all possible combinations prior to (p-1)(m) iterations.

---

* For an m x n assignment problem where n > m, the number of possible solutions is $\frac{(n-1)(n-1)!}{(n-m)!}$

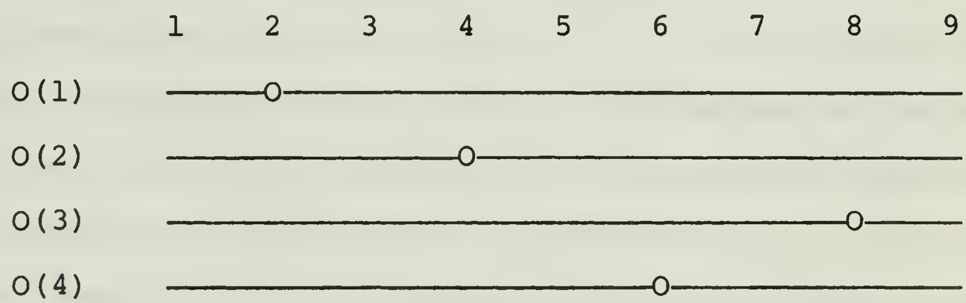
For an n x n assignment problem, the number of possible solutions is $\frac{(n-1)(n-1)!}{2}$

E. Determination of the Assignment Time

As was inferred previously, a detailer may decide that the total amount of extension time required or the officers that would necessarily need be extended to implement the best numerically preferred solution is not justifiable. Needed then is an efficient method to present to the detailer the minimum amount of extension time required to accomplish the numerically preferred solution or any of the lesser numerically preferred solutions. We remind the reader that the phrase "preferred solutions" is reserved for the numerically preferred solution that has, in conjunction with its associated minimum extension time, the most acceptable characteristics in the mind of the decision maker.

Refer now to Figure 8 which has only the projected rotation dates [O] plotted for each officer, one through four.

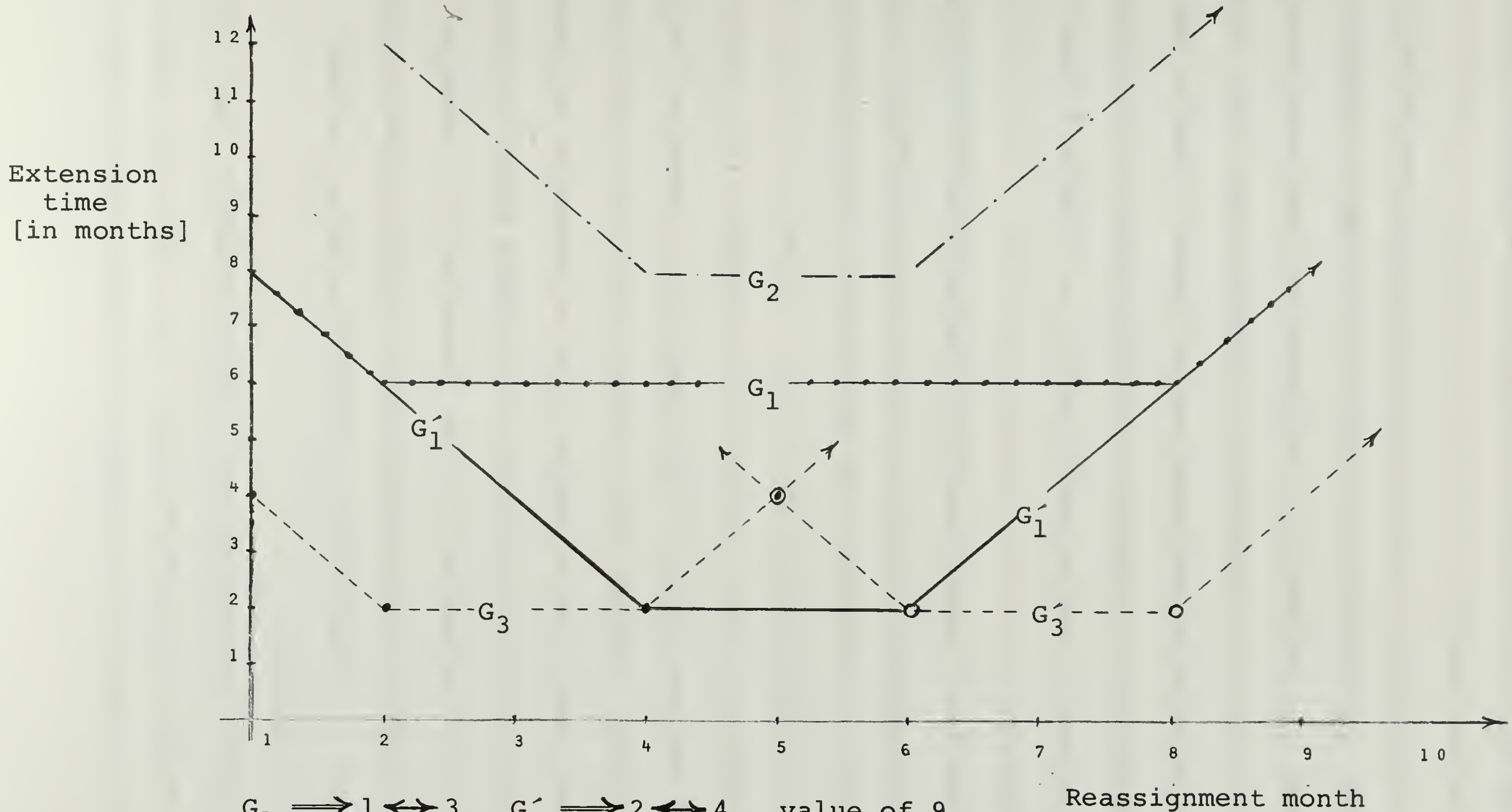
FIGURE 8



An *extension* as used in the following discussion will constitute either *a lengthening* of an officer's tour *past* his projected rotation date or *a shortening* resulting in reassignment *prior* to his projected rotation date.

A plot, Figure 9, was then made for three (the best, 5th best, and 9th best) numerically preferred solutions to indicate the amount of extension time required to effect each

FIGURE 9



$G_1 \Longrightarrow 1 \longleftrightarrow 3 \quad G_1' \Longrightarrow 2 \longleftrightarrow 4$ value of 9

$G_2 \Longrightarrow 1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4 \longrightarrow 1$ value of 14

$G_3 \Longrightarrow 1 \longleftrightarrow 2 \quad G_3' \Longrightarrow 3 \longleftrightarrow 4$ value of 18 solution as a function of the month in which the solution might be executed.

For example, if the numerically preferred solution (a_3 b_4 c_1 d_2) which is (O1 $\rightarrow$ B3, O2 $\rightarrow$ B4, O3 $\rightarrow$ B1, and O4 $\rightarrow$ B2) was implemented, officers one and three would necessarily have to be reassigned in the same month and the same comment applies to officers two and four. Then as each month is considered as a candidate for implementing the numerically preferred solution, graphs G_1 and G_1' of Figure 9 are generated.

Take for example graph G_1, the ordinate value at month 1 was generated by adding the interval (one month) from month 1 to O(1)'s P.R.D. to the interval (seven months) from month 1 to O(3)'s P.R.D.

Graphs G_1' and G_1, which show extension time required to execute the two parts of the numerically preferred solution, respectively indicate that months four, five or six require the least amount of extension time to reassign officers two and four; and that any month two through eight is best, as far as total extension time is concerned, to reassign officers one and three. The minimum total extension time needed to implement the numerically preferred solution is then $2 + 6 = 8$ months.

Graph G_2 indicates the minimum extension time, also eight months, required to effect the fifth best numerically preferred solution is incurred in either months four, five or six.

Summing the minimum points on Graph G_3' and G_3 one finds that the worst numerically preferred solution ($1 \leftrightarrow 2$, $3 \leftrightarrow 4$) has an extension requirement of only four months.

The best numerically preferred solution then clearly dominates the fifth best numerically preferred solution since it has a lower total assignment value (9 compared to 14) and the same total extension time (eight months). But does either the numerically preferred or fifth numerically preferred solution dominate the ninth best (worst) numerically preferred solution? Both have smaller total assignment values and <u>larger</u> total required extension times. The preferred solution in these cases can only be resolved in the mind of the detailer as he subjectively weighs trade-offs between total required extension times and total assignment values.

CHAPTER III

THE RELATIVE VALUE MODEL

A. Introduction

Some quantifiable "value" or measure must be found if the model developed in the preceding chapter is to be of practical use. Development of such a measure must consider: (1) the role of the measure in the decision process and (2) the mathematical properties required by the analytical scheme.

The ideal measure, of course, would be one which would functionally relate the overall effectiveness of the Navy to the various assignments possible. This would allow assignments to be determined on the basis of maximum overall Naval effectiveness. Such a measure is at present well beyond the state of the art.

However, there is an approximation available which meets the joint requirements. From the assignment point of view any man-to-billet comparison must consider two criteria: (1) how well do the officer's qualifications satisfy the billet requirements? and (2) how well does the billet satisfy the officer's professional development requirements?

The second criterion is assumed to be satisfied when the assignment officer determines which billets an officer can be considered for. The following development will be quite general, however, and the preceding statement is not meant to imply that the second criterion need be satisfied in this manner. Indeed, an assignment officer's judgment could be contained in the individual qualification vectors which will now be discussed.

To determine the extent that an officer's qualifications meet a specific billet requirement it should be first noted that his qualifications have meaning only in the context of satisfying a specific billet's requirement. For example, an aviator's flight qualifications have little meaning unless he is being ordered to a flight billet. It should also be clear that all qualifications of a billet are not of equal importance, i.e., a billet qualification such as "previous experience" might be more important than "formal training". For the above reasons, the relative value measure chosen to meet the first criteria will be established by a two-step process as follows: (1) determine the weighted requirements of the billet and (2) compute in an officer-to-billet comparison the extent that the officer fulfills such requirements. The development will assume that a linear approximation is sufficient to describe the relative value of not meeting a particular qualification, i.e., it is assumed that if a two-thousand hour, total flight time requirement has been established, that a man with one-thousand flight hours partially fulfills the requirement. The value measure so defined indicates to what extent the officer's qualifications have met the weighted billet requirements.

This development is equivalent to King's[6] "pure programming approach" and conceptually satisfies the ratio-scale measure requirements necessary in the allocation model. This approach has been chosen since it appears to be most amenable to computer application on a large-scale basis. Other measures could be developed and probably will be. The important consideration, of course, is that the assignments generated with the measure do in fact reflect how well the officers assigned meet the billet requirements. Measures and methods applicable to this determination will be included in Section B.

B. Development

Considering a general m x n problem of this type, let $A = (a_{ij})$ represent a matrix such as that in Table 3 and the problem may be formally stated as one of selecting an m x n matrix $Y = (y_{ij})$ to satisfy the following:

$$\text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} y_{ij} \qquad \text{(III-1)}$$

$$\sum_{i=1}^{m} y_{ij} = 1, \ \forall j \qquad \text{(III-2)}$$

$$\sum_{j=1}^{n} y_{ij} = 1, \ \forall i \qquad \text{(III-3)}$$

$$y_{ij} \in (0,1), \ \forall i,j \qquad \text{(III-4)}$$

Let the vectors Q_i (i=1, ..., m) be the qualifications (resources) possessed by each officer O_i (i=1, ..., m); these qualifications make up the p elements $(q_{i1}, q_{i2} \ldots, q_{ip})$ of each Q_i vector. Let the vectors B_j (j, ..., n) be the desirable levels of qualifications necessary (requirements) for successful performance of billet B_j, these requirements make up the p elements $(b_{j1}, b_{j2}, \ldots, b_{jp})$ of each B_j vector. The assignment problem is then one of allocating the resources (Q_i) to the desired requirements (B_j).

If V_{ijr} is the relative value associated with the deviation $(b_{jr} - q_{ir})$ from the desired requirement for the $r^{\underline{th}}$ qualification in B_j, the problem is one of determining $Y = [Y_{ij}]$ as defined in equations III-1 through III-4 such that III-5 is satisfied.

$$\text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{n=1}^{p} V_{ijr} Y_{ij} \qquad \text{(III-5)}$$

It is apparent that the determination of the V_{ijr} would, in practice, be no easy task. Reasonable approximations will make this approach more applicable, e.g., the criterion (III-5) might be approximated as (III-6)

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{p} v_{jr}\, x_{ijr}\, y_{ij} \qquad \text{(III-6)}$$

where

$$x_{ijr} = \begin{cases} b_{jr} - q_{ir} & \text{if } b_{jr} - q_{ir} > 0 \\ 0 & \text{if } b_{jr} - q_{ir} \leq 0 \end{cases}$$

and

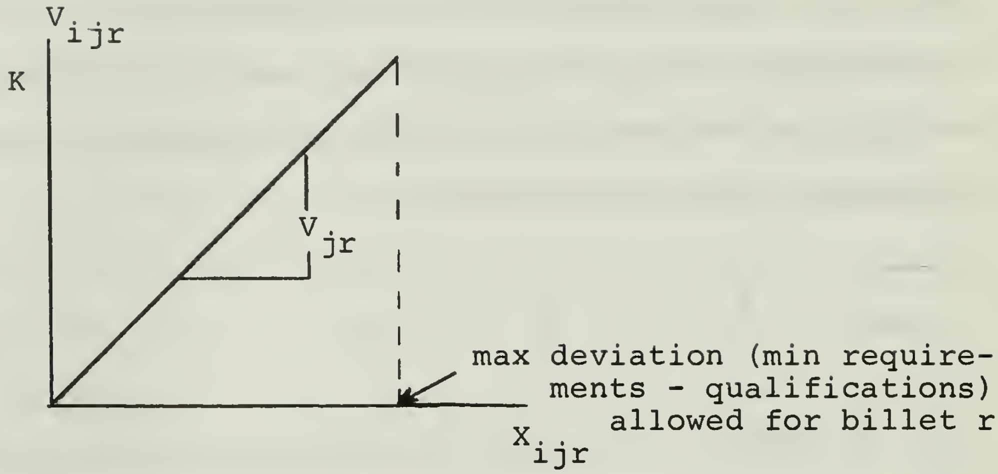
$v_{jr} \equiv$ the relative value of a unit deviation, x_{ijr} $(\forall_i)$, from the desired requirement of resource r in B_j.

This criterion (III-6) is a linear approximation to (III-5) i.e.,

$$V_{ijr} \simeq v_{jr}\, x_{ijr} \;\forall_{i,j,r}$$

The relative value functions are all assumed to be of the form as given in Figure 10.

FIGURE 10



The p elements of a requirements vector (B_j) can also be weighted for each billet. Let d_{jr} represent the relative weight (importance) given a particular qualification of a billet. Then

$$d_{jr} \geq 0 \quad \forall r,j \text{ and } \sum_r d_{jr} = 1, \forall j \qquad \text{(III-8)}$$

Set an upper bound, K, on the worst relative value than can be achieved by not meeting the minimum on requirement r. In particular let V_{ijr} be defined on $[0, K] \forall i,j,r$

then

$$a_{ij} = \sum_{r=1}^{p} d'_{jr} v_{jr} x_{ijr} \simeq \sum_{r=1}^{p} d'_{jr} V_{ijr}, \forall i,j \qquad \text{(III-9)}$$

$$\text{and } d'_{jr} = \frac{\max a_{ij}}{K} (d_{jr}) \qquad \text{(III-10)}$$

d'_{jr} is a factor to normalize $0 \leq a_{ij} \leq \max a_{ij}$

The a_{ij} are then finally the elements of the relative value matrix.

The following calculation of a_{11} is an attempt to clarify the notation and definitions of this section.

There are three billets (B1, B2, and B3, i.e., n=3) and three officers (O1, O2, O3, i.e., m=3) available in the time horizon under consideration. Let K = 25, $\forall_{j,r}$. Therefore, max a_{ij} = 3 x 25 = 75.

Listed below are the requirements for each billet.

	Requirements Desired	Min. Desired Req's.
Billet 1 (i.e., j=1)	b_{11} = 1500 hrs. prop. flight time.	500 hrs. prop. flight time
	b_{12} = LT	LTJG $\leq$ rank $\leq$ LCDR
	b_{13} = S code in E.E.	1 previous tour related to electronics
Billet 2 (i.e., j=2)	b_{21} = 100 hrs. prop. flight time.	25 hrs. prop. flight time
	b_{22} = LTJG	ENS $\leq$ rank $\leq$ LT
	b_{23} = graduate of Electronics course, Memphis	0
Billet 3 (i.e., j=3)	b_{31} = 3000 hrs. total flight time.	1500 hrs. total flight time
	b_{32} = LCDR	LT $\leq$ rank $\leq$ CDR
	b_{33} = P code in E.E.	2 previous tours related to electronics or graduate of electronics course, Memphis

Qualifications

Officer 1 (i.e., i=1)	q_{11} = 1000 hrs. prop. flt. time; 1500 total
	q_{12} = LT, 1 1/2 years in pay grade
	q_{13} = graduate of E.E. course, Memphis
Officer 2 (i.e., i=2)	q_{21} = 2000 hrs. prop. flt. time; 3500 total
	q_{22} = LCDR, 1 year in pay grade
	q_{23} = 1 previous tour as electronics officer, another as Electronics Warfare Officer
Officer 3 (i.e., i=3)	q_{31} = 125 hrs. prop. flt. time; 125 total
	q_{32} = LTJG, 6 months in pay grade
	q_{33} = S code in E.E.

The assignment officer considers for B(1) the requirement regarding flight time to be twice as important a factor for successful performance as the requirement regarding rank and six times as important as the requirement to do with educational background.

Therefore $d_{11} = .6$, $d_{12} = .3$, and $d_{13} = .1$ in keeping with (III-8).

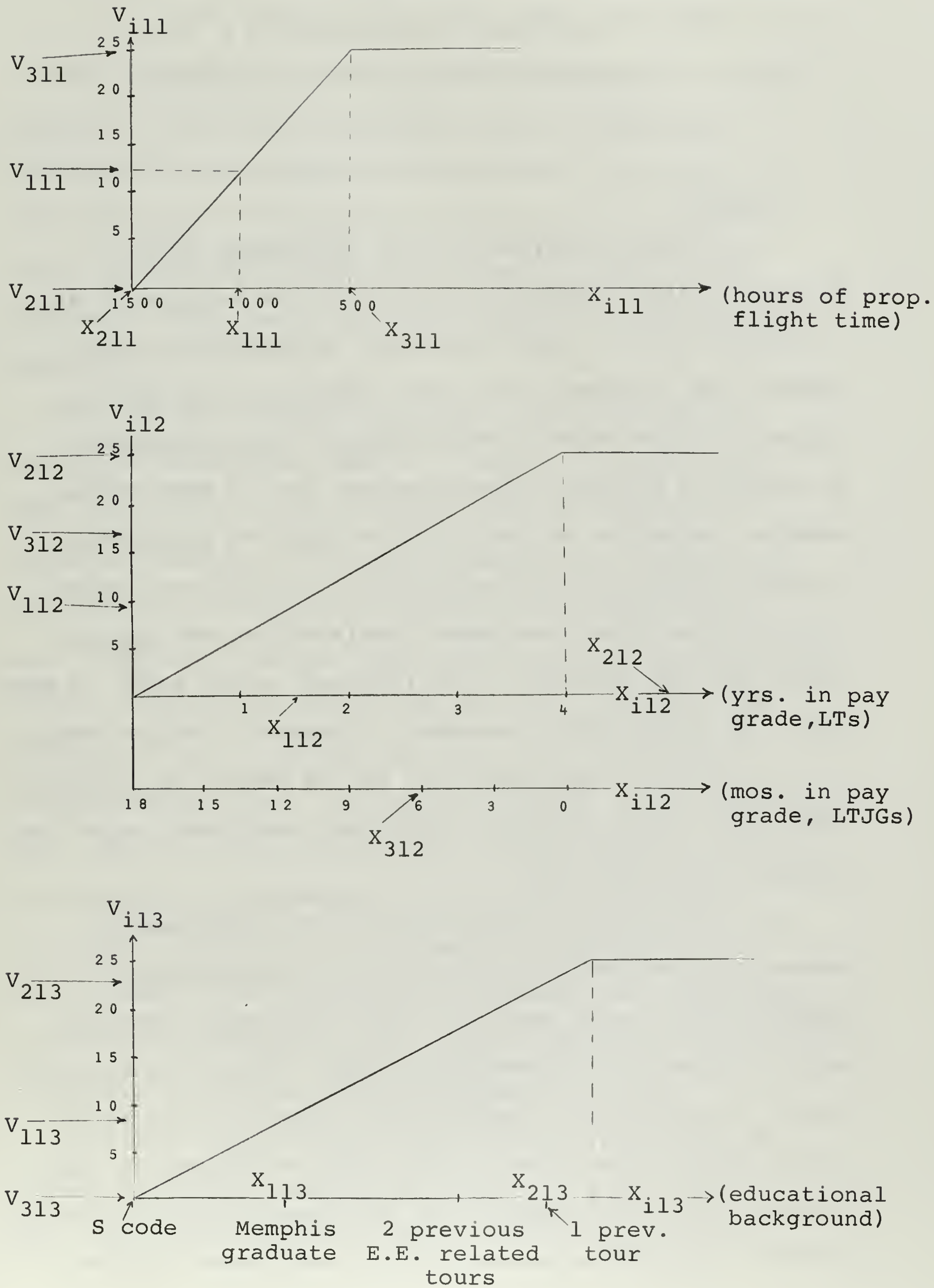
Then from (III-10):

$$d'_{11} = \frac{75}{25} \times .6 = 1.8, \quad d'_{12} = .9, \quad \text{and } d'_{13} = .3$$

The V_{ijr} as taken from Figure 11 are

$$V_{111} = 25 \qquad V_{112} = 13 \qquad V_{113} = 16$$

FIGURE 11



Vi11
25
20
15
10
5
V311
V111
V211
1500
1000
500
X211
X111
X311
Xi11
(hours of prop. flight time)
Vi12
V212
V312
V112
1
2
3
4
X212
Xi12
X112
(yrs. in pay grade,LTs)
18
15
12
9
6
3
0
X312
(mos. in pay grade, LTJGs)
Vi13
V213
V113
V313
X113
X213
Xi13
(educational background)
S code
Memphis graduate
2 previous E.E. related tours
1 prev. tour

and then from Equation (III-9)

$$a_{11} = d'_{11} V_{111} + d'_{12} V_{112} + d'_{13} V_{113}$$

$$= (1.8)(12.5) + (.9)(9.3) + (.3)(8)$$

$$a_{11} \simeq 33$$

C. Summary

The ultimate purpose of the "assignment value" defined in this chapter is its use in the assignment or allocation model. Since the model is designed to complement the assignment officer's judgment, and not to replace it, the value measure desired need not predict an officer's probable effectiveness, but in some manner measure his degree of qualification for the assignment. It must certainly be recognized that many factors which influence the final assignment decision are not quantifiable and too variable to be included in any model if they were. It seems quite reasonable, however, that the basis of any assignment decision must be the degree of qualification for the assignment. This the "assignment value" as defined should provide.

The development of this particular value measure should provide additional benefits to the assignment officer, for a major portion of his assignment investigation is devoted to determining each officer's qualifications. Use of such a qualification index could substantially reduce this effort. The value as defined in this chapter seems quite appropriate for computer use, and will provide a meaningful measure to the assignment officer.

CHAPTER IV

SUMMARY AND CONCLUSIONS

The effectiveness of the officer distribution system depends on the daily assignment decisions made by the various grade assignment officers within the Bureau of Naval Personnel. The purpose of this study was to design and develop a solution procedure which would complement their experience and judgment in making such decisions.

From the most general analysis of the assignment cycle, the assignment officer's problem at any given time was shown to indeed be the familiar "optimal assignment problem" of linear programming. Because of the factorial nature of the set of "possible assignment decisions" and the large number of officers involved in most practical problems, it was clear that present assignment-decisions must be made on the basis of a reduced decision set. Is the set reduced to the preferred set? The set of "possible assignments" could be reduced by eliminating assignments on the basis of various assignment criteria or more simply by assigning the problem away, i.e., consider one officer and assign him, a second and assign him, etc. Such a sequential procedure, of course, rarely provides an optimal solution.

The study investigated a sequential solution of the officer assignment problem on a continuing time basis. Such a procedure of course was still sub-optimization.

Intuitively, it is evident that the larger the set considered becomes, at each sequential step, the better the solution is. This is especially true in officer distribution since officers are made available for assignment on the basis of pre-established rotation dates and more officers are considered as the time interval is increased.

If assignments must be made in some time interval about their rotation dates, officers can only be considered for billets available in the interval. Since an officer's qualifications satisfy a billet's requirements if and only if he is assigned to the billet, effective assignments can be realized only when the billet and officer are jointly available. It is apparent that "when" an officer is assigned is as equally important as "how" the assignment is made. It can be concluded that an increased consideration period could provide more effective assignments.

However, this study recognized that some means such as the present rotation system must be utilized to insure orderly rotation although this implies a reduced interval of consideration. The two objectives; orderly rotation and effective assignments are normally conflicting. In addition, increasing the size of the problem increases its complexity in a factorial manner.

The assignment model developed in this thesis could provide the assignment officer the means with which to consider greater numbers of assignments effectively. This, of course, implies an increased assignment horizon. The model employs a network theory formulation and solution with the "out-of-kilter algorithm" of Ford and Fulkerson. A value measure based on King's "Pure" Programming Approach, which indicates the degree to which each officer's qualifications meet the weighted billet's requirements is utilized for solution. The solution of the assignment problem for a time period provides: (1) any set of solutions desired, by an iterative procedure from the numerically preferred solution, (2) the time to make the assignment for any given solution. This time is in effect that date or dates within the interval under consideration which requires the minimum movement of pre-established rotation dates.

In summary, the solution procedure developed provides the following major advantages:

(1) It can be used to provide any set of the ordered quantative solutions desired.

(2) It is a flexible solution procedure with which to readily respond to an altered assignment environment.

(3) It permits each officer to be considered for more billets, by providing the assignment officers with time/assignment effectiveness trade-offs.

(4) It provides the assignment/placement officer team with specific information as to when and how a future billet requirement can best be met.

It is concluded that the Bureau of Naval Personnel might want to consider the practical utilization of this model for a pilot evaluation on a single rank desk. The officer assignment problem appears too complex in scope for a human to comprehend and too variable for a machine to be of use. Effective assignment decisions require the use of both: a computer to reduce the problem to the significant assignment alternatives; and a detailer's judgment and experience to make the final assignment decision.

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13. ABSTRACT

The effectiveness of the officer distribution system of the Navy is strongly dependent on the assignment officers' daily assignment decisions. The officer assignment problem is to determine the optimal assignment of officers to billets on a continuing time basis. A procedure is developed in this study by which ranked assignment alternatives can be provided the assignment officer to assist him in making his decisions. The ranking of the alternatives is based on an index or value measure developed from the quantifiable assignment information. The assignment alternatives are developed for a specified assignment period of interest and represent trade-offs between feasible assignments and times of assignment. The procedure makes long range assignment planning feasible by reducing the problem to one of manageable size for the assignment officer's consideration.

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